Exercise 7

The radius of a spherical ball is increasing at a rate of 2 cm/min. At what rate is the surface area of the ball increasing when the radius is 8 cm?

Solution

The surface area of a sphere with radius r is

$$S = 4\pi r^2.$$

Differentiate both sides with respect to t, using the chain rule on the right side.

$$\frac{d}{dt}(S) = \frac{d}{dt}(4\pi r^2)$$

$$\frac{dS}{dt} = (8\pi r) \cdot \frac{dr}{dt}$$

The radius is increasing by 2 centimeters per minute, so $dr/dt = 2$ cm/min. Therefore, when the radius is 8 cm, the rate that the surface area is increasing is

$$\left.\frac{dS}{dt}\right|_{r=8} = 8\pi(8)(2) = 128\pi \, \frac{\text{cm}^2}{\text{min}}.$$